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Reinforced Concrete Corners and Joints Subjected to Bending Moment[a]

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Introduction

In simple structural members, such as columns and beams, the principal locations of the rinforcement are obvious. The aim of research in these areas has been to predict accurately the strength and deformation properties of these members. On the other hand, reinforcement detailing is not obvious for corners and joints that occur in frame structures. Therefore, extensive experimental work has been conducted to study the modes of action of reinforcement alternatives when these structural elements are loaded to failure. In the course of the tests, serious deficiencies were often found in reinforcement detailing.

Work on this investigation was started in 1965 at the Division of Concrete Structures, Chalmers University of Technology, Göteborg, Sweden. It was financed by the Swedish Council for Building Research and the Swedish Road Board. The results have been published gradually in several test reports (5,6,7,8). In 1973 the investigation was published as a final report (4) of which this paper is a brief summary.

Aim of Investigation.—The primary objective of the investigation is to develop simple and rational methods for design of the reinforcement in corners and joints subject to bending moments. The reinforcement details must satisfy the fundamental requirements of strength, limited cracking, ductility, and simplicity of construction. This investigation comprises 78 corners and joints with different dimensions and reinforcement under monotonic one-direction loading. The main emphasis is placed on the study of frame corners subjected to positive bending moments, i.e., moments that cause tensile stresses on the inside of the corner

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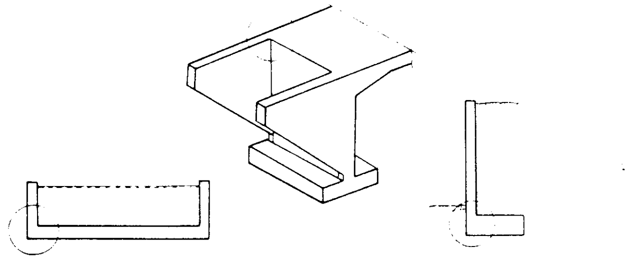


FIG. 1.—Examples of Corners Subjected to Positive Moments

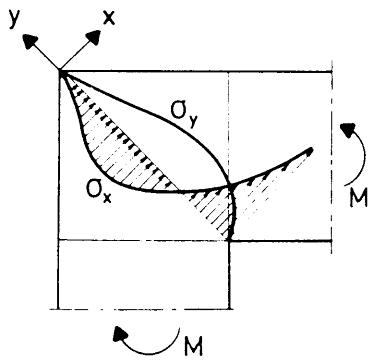


FIG. 2.—Stresses σ_x and σ_y along Corner Diagonal

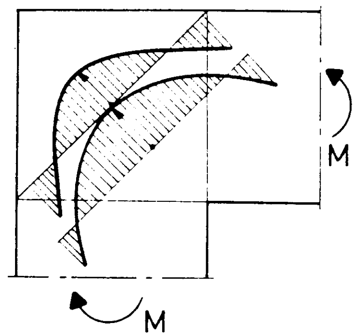


FIG. 3.—Tensile Stresses across Corner Sections Considered Parabolically Distributed

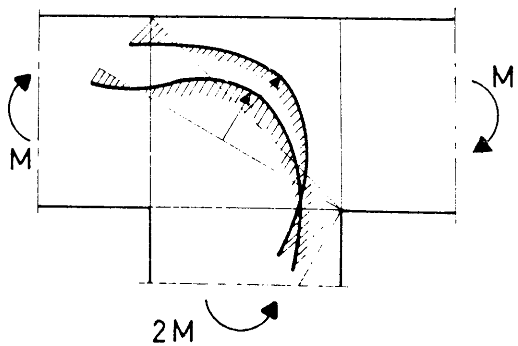


FIG. 4.—Tensile Stress Distributions across Joint Sections Considered Parabolically Distributed

(see Fig. 1). Negative corner moments cause less detailing problems. There may, however, be a risk of splitting crack failure under the reinforcement bends or anchorage failure (4). These modes of failure are not dealt with in this paper.

Requirements Regarding Reinforcement Layout.—The reinforcement should be placed in such a way that the joints meet certain basic requirements:

1. The joint should be capable of resisting a moment at least as large as the calculated failure moment in adjacent cross sections, i.e., begin yielding in the beam reinforcement. Consequently, failure in the joint does not occur and the structure is able to develop its computed strength.
2. If it is not possible to meet the first requirement, then the reinforcement layout should satisfy a second requirement. The joint should have the necessary ductility to carry large deformations so that redistribution of forces in the structure will be possible without brittle failures of the joints. In statically determinate structures, such as bridge abutments, retaining walls, and open channels, there is no redistribution of moments to adjacent structural elements. In this case the strength of the corner is, critical for the integrity of the structure.
3. Cracks form on the inside of corners which are subject to tension. The widths of corner cracks should, therefore, be limited to an acceptable value in the working range.
4. The reinforcement should be easy to fabricate and position. Corners and joints are often of considerable width. For such joints, details, which include stirrups, may be difficult to place in practice, and care must be taken that the reinforcement does not seriously disturb the casting of the concrete. Thus, the work concentrated on finding reinforcement layouts that made it possible to avoid stirrups in corners and joints.

Stress Distribution According to Theory of Elasticity

The state of stress in corners and joints as calculated by the theory of elasticity is valid only before cracking occurs (Stage I). After cracking and at later stages (Stages II and III), the joint acts as a composite structure made up by the reinforcement and the concrete. Thus the analysis of joint behavior is far more complicated than that of homogeneous bodies.

Despite the fact that the results provided by the theory of elasticity are valid only prior to cracking, they help to indicate where tensile stresses occur. From an elastic analysis some guidance regarding the location of tension reinforcement in corners and joints is provided. Results from analyses of this kind are given in Figs. 2-4.

Fig. 2 shows the stress distribution along the diagonal in a corner subjected to positive moment. The bending stress, σ_x, exhibits a peak tension at the inside of the corner, which explains why corner cracks occur for quite small loads. The tensile stresses, σ_y, cause a diagonal crack across the corner which results in sudden failure unless reinforcement is provided. These tensile stresses may be considered parabolically distributed perpendicular to the joint diagonal (see Fig. 3).

Fig. 4 shows stresses in a T-joint subjected to bending moments. The tensile stresses across the joint sections may again be considered parabolically distributed. If the layout of the reinforcement is unsuitable, the tensile stresses may cause a diagonal crack in the joint which results in sudden failure.

Causes of Failure of Corners and Joints in Reinforced Concrete

In corners and joints, failure may occur for several reasons. Five causes of failure are explained in Ref. 4, i.e., diagonal tension crack failure, splitting crack failure, failure due primarily to yielding of the reinforcement, anchorage

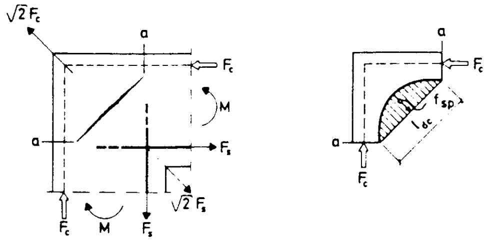


FIG. 5.—Truss Idealization of Corner Subjected to Positive Moment

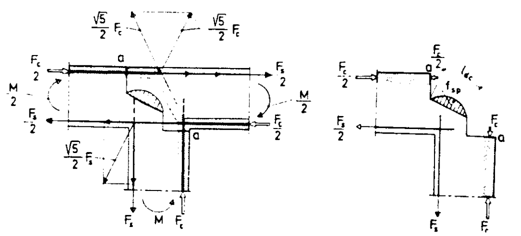


FIG. 6.—Truss Idealization of T-Joint Subjected to Bending Moment

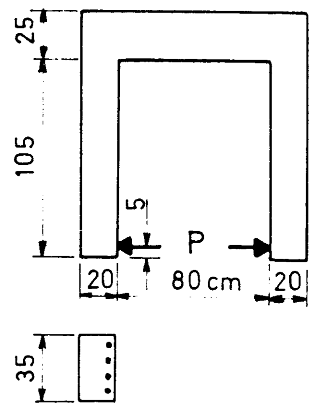


FIG. 7.—Dimensions and Method of Loading of Test Model (1 in. = 25.4 mm)

failure, and failure due primarily to crushing of the concrete. Of these, diagonal tension crack failure caused by bending moment has received the least attention, although it has been found to be a common cause of failure in corners and joints.

Diagonal Tension Crack Failure Caused by Bending Moment.—The occurrence of a diagonal crack gives rise to sudden failure. One of the main problems in this work, therefore, has been to prevent this type of failure by suitable reinforcement detailing.

The diagonal tension crack failure is analyzed with the aid of truss models in which the reinforcement is regarded as the tension bar and the zones of compression in the concrete are the compression struts. In traditional truss models, the forces in these struts satisfy equilibrium conditions. In the models used in this case, the portion of the concrete in tension also contributes to the equilibrium. The models, which approximately reproduce the system of forces, are verified experimentally.

In the analysis, the concrete tensile strength, f_{sp}, as determined by splitting tensile tests, is used. This splitting strength value varies somewhat from the pure tensile strength of the concrete, and consequently is an approximation. The choice of the position and length of the diagonal crack may, in practical applications, be a source of uncertainty.

Corner Subjected to Positive Moment.—Fig. 5 shows the system of forces in a corner subjected to a positive moment. The resultant tensile force across the diagonal is $\sqrt{2}\,F_s = \sqrt{2}\,F_c$. According to the theory of elasticity, the tensile force just before cracking may be considered to have a parabolic distribution (see Fig. 3):

$$\sqrt{2}\,F_s = \frac{2}{3} f_{sp}\, b\, l_{dc} \qquad (1)$$

in which f_{sp} = the tensile strength of the concrete; b = the width of the corner; and l_{dc} = the length of the tensile stress distribution = length of the diagonal crack (compare Fig. 3). The bending moment on the corner is $M = F_s z$, in which z = the lever arm = $0.8\,d$ (acceptable assumption); d = the effective depth of the member framing in the corner or joint; and

$$M = F_s 0.8\,d \qquad (2)$$

Equating Eq. 1 and 2 gives

$$M_{dc} = \frac{\sqrt{2}}{3} 0.8\,d\,b\,l_{dc} f_{sp} = 0.38\;d\,b\,l_{dc} f_{sp} \qquad (3)$$

T-Joint Subjected to Bending Moment.—A T-joint subjected to bending moment and with all members framing into the joint of the same depth (Fig. 6) will be studied with the aid of a truss model.

The compressive forces on the joint are assumed to be balanced by an inclined compressive strut. The tensile force from the main reinforcement tensile forces resolved across the diagonal is $(\sqrt{5}/2)F_s$ according to the assumptions made in Fig. 6. On the basis of an assumption of the stress distribution across the diagonal, it is possible to predict the moment, M_{dc}, at which the first crack occurs. According to Fig. 4, it is reasonable to assume that the tensile force is parabolically distributed across the diagonal before the occurrence of the crack:

$$\frac{\sqrt{5}}{2} F_s = \frac{2}{3} f_{sp}\, b\, l_{dc} \qquad (4)$$

The bending moment on the joint is

$$M = F_s z = F_s 0.8\, d \qquad (5)$$

Eqs. 4 and 5 give

$$M_{dc} = \frac{4}{3\sqrt{5}} 0.8\, d\, b\, l_{dc} f_{sp} = 0.48\, d\, b\, l_{dc} f_{sp} \qquad (6)$$

For corners and joints with other geometrical dimensions and combinations of moments, it is possible to derive similar equations. However, they are more complicated.

Experimental Investigation of Corners Subjected to Positive Moment

A preliminary investigation (specimens U1, U2, and U3) carried out in 1965 (5) found that the reinforcement detail used in wing wall corners of bridge abutments was incorrectly designed and resulted in failure at about one-third of the intended strength. These details were commonly used in Sweden and elsewhere. Therefore, an investigation was started in order to study systematically alternative methods of placing the reinforcement.

Tests on Some Simple Reinforcement Details.—Specimens with dimensions as shown in Fig. 7 were used for the tests. The specimen reproduced well the actual load conditions on practical corners subjected to positive moments. The length of the leg was made large enough so that flexural failure next to the beam could occur without any risk of the load, P, causing shear failure.

Several test series were performed and will be described subsequently. Four simple reinforcement layouts were chosen as an introduction. Three tests were carried out in every detail with reinforcement percentages in the leg members of $\rho = 0.5\%$, 0.75%, and 1.1%. The reinforcement percentage is defined as $\rho = A_s / bd$, in which A_s = the area of the main tensile reinforcement. The tests were carried out in 1965-1966 and were described extensively in a test report (6).

Reinforcement in Accordance with Earlier Practice.—The first reinforcement detail studied, specimens U20, U21, and U22, was reinforced in the same way as the pilot tests. On loading to failure, the same type of cracking occurred in the corners (see Fig. 8). The portion of the corner outside the bent reinforcement was pushed off due to diagonal tension crack failure. Failure occurred suddenly with the ultimate load capacity in these three tests remaining about constant and independent of the percentage of reinforcement.

Reinforcement Detail with Hairpins.—The next corner detail studied in specimens U14, U15, and U16 was reinforced with bent bars in the form of hairpins. The planes of the hooks were perpendicular to the top surface of the specimen. Failure was again caused by the occurrence of a diagonal crack that caused the portion of the corner outside a line through the anchored bends to be pushed off (see Fig. 9).

Reinforcement with Loops.—An old way of placing the reinforcement in a corner, which is often used, is to form it into a loop. This arrangement was tested in specimens U11, U12, and U13. At application of load, a wide corner crack occurred which under further loading divided and followed the reinforcement loop out into the corner until the crack surrounded the whole loop (see Fig. 10). At an increased load level the portion of the corner outside the loops was finally pushed off.

An alternative to the loop is the use of two bent bars. An advantage of this method is that in the case of connecting beam sections with different depths, two bent bars will enclose the corner with reinforcement better than the loop. The three specimens tested, U23, U24, and U25, got somewhat higher efficiency than corresponding loops.

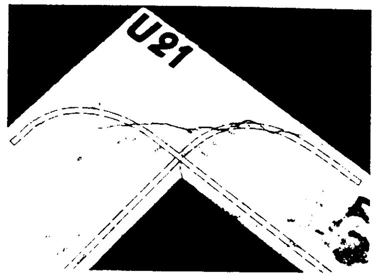


FIG. 8.—Corner Subjected to Positive Moment (Tension on Inside; Reinforcement Drawn on Surface; Diagonal Crack Caused Sudden Failure)

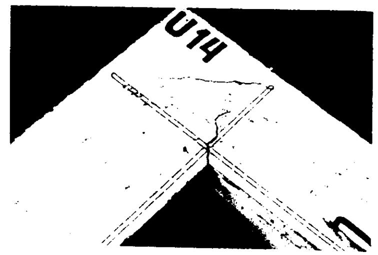


FIG. 9.—Corner Reinforced with Hairpins after Diagonal Tension Crack Failure

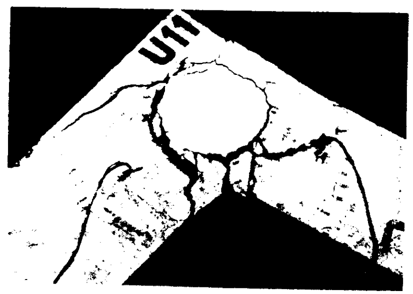


FIG. 10.—Corner Reinforced with Loops (At Failure, Cracks Run Along Reinforcement Loops and Concrete Outside Reinforcement Is Pushed Off)

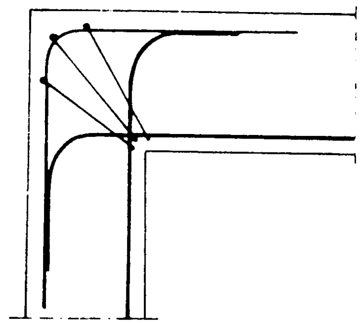


FIG. 11.—Corner Reinforcement—Detail U28

Corner Reinforcement Details with Stirrups.—Corner reinforcement details with the addition of stirrups are often referred to in available literature. Fig. 11 shows an example. In an attempt to resist the tensile stresses, σ_y, and thereby prevent spalling of the corner as shown in Fig. 8 (specimen U21), the joint was augmented by stirrup reinforcement. The stirrups were difficult to place and the presence of all the bars in the corner made it difficult to cast the concrete. On application of the load, failure occurred in the corner (Fig. 12). Failure was not preceded by reinforcement yielding in the corner, but was caused by an insufficient arrangement of the reinforcement (7). The stirrups

FIG. 12.—Detail of Corner Crack and Diagonal Cracks after Failure

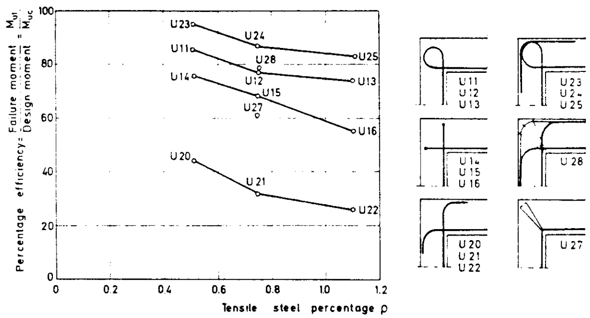


FIG. 13.—Corner Efficiency M_{ut}/M_{uc} as Function of Reinforcement Percentage ρ for Some Simple Reinforcement Proposals

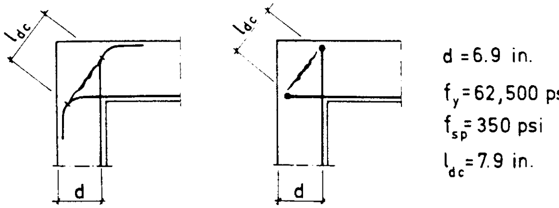


FIG. 14.—Positions of Diagonal Cracks and Approximative Data for Two Corner Reinforcement Details (1 in. = 25.4 mm; 1 psi = 6.9 kN/m²)

increased the strength and ductility of the corner. However, the use of stirrups in corners and joints of long length, as between walls and slabs, is not usually a technically desirable solution due to the practical difficulties of placing and casting.

Comparison of Reinforcement Arrangements Tested.—The results of the first reinforcement details tested are compared in Fig. 13. The diagram shows the

TABLE 1.—Diagonal Cracking Moments

Specimen number (1)	Concrete tensile strength, f_{sp}, in pounds per square inch (2)	M_{dc}, in foot-pounds: Calculated (3)	M_{dc}, in foot-pounds: Observed (4)	Observed/calculated (5)
U1	380	34,720	40,500	1.17
U2	370	23,150	24,190	1.04
U3	405	34,000	33,700	0.99
U20	310	7,300	7,410	1.01
U21	390	8,900	7,160	0.81
U22	345	7,890	8,790	1.11
U14	385	9,190	12,770	1.39

Note: 1 psi = 6.9 kN/m².

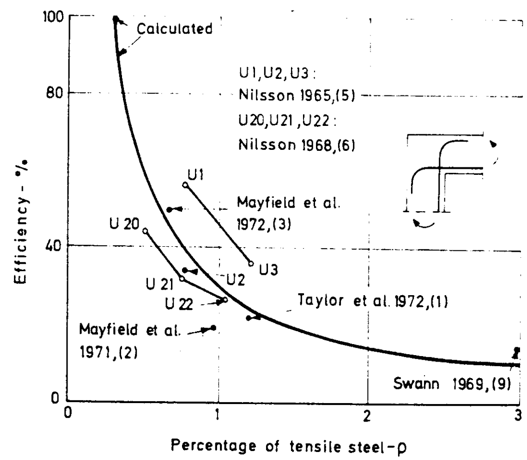


FIG. 15.—Efficiency of Corner Detail (Solid Curve Shows Calculated Efficiency; Good Agreement with Available Test Results)

corner efficiency, M_{ut}/M_{uc}, as a function of the reinforcement percentage, ρ, M_{ut} = the ultimate moment measured by test; and M_{uc} = the ultimate moment calculated from bending theory. When $M_{ut}/M_{uc} \geq 100\%$ the corner is at least as strong as the connecting sections.

Application of Diagonal Tension Crack Failure Analysis.—In the corner reinforcement detail series U1-U3 and U20-U22 (with the same reinforcement arrangement) as well as the specimens in the series U14-U16, diagonal tension cracks caused sudden failure. The moment, M_{dc}, at which diagonal tension crack failure occurs was derived previously with the help of the truss analogy, i.e., Eq. 3. This expression will be applied to these tests. In the equation, l_{dc} is the length of the tensile stress distribution before cracking according to the theory of elasticity and the stresses are assumed to be parabolically distributed (see Fig. 3). Positions of diagonal tension cracks and approximate values for the calculation of M_{dc} are given in Fig. 14. Table 1 summarizes calculated and observed values of M_{dc}. The agreement is acceptable.

The cracking moment, M_{dc}, depends on the tensile strength of the concrete, and therefore the diagonal tension crack failure has a brittle nature. If the joint is to be capable of taking plastic deformations, the tensile reinforcement should reach its yield stress before the occurrence of the diagonal crack. This means that the tensile force, F_s, at which the diagonal tension crack failure occurs, must satisfy the condition

$$F_s \geq A_s f_y \qquad (7)$$

Inserting this in the equation for the occurrence of the diagonal tension crack, Eq. 1, we obtain

$$A_s f_y \leq \frac{\sqrt{2}}{3} f_{sp}\, b\, l_{dc} \qquad (8)$$

Inserting $A_s = \rho\, b\, d$, we obtain

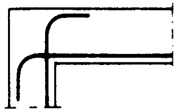
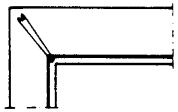
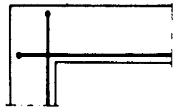
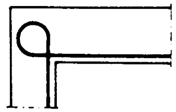
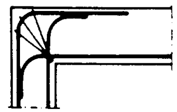
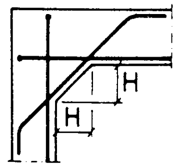
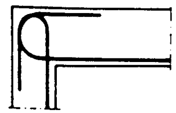
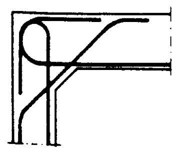
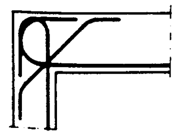
$$\rho \leq \frac{\sqrt{2}}{3}\, \frac{f_{sp}}{f_y}\, \frac{l_{dc}}{d} \qquad (9)$$

i.e., if the reinforcement percentage, ρ, satisfies the condition according to Eq. 9, the tensile reinforcement will yield before the diagonal crack occurs (4).

When Eq. 9 is applied to corners with dimensions, concrete, and steel properties according to tests U20, U21, and U22, diagonal tension cracking failure will not occur if $\rho \leq 0.3\%$. On the assumption that the diagonal cracking failure moment, M_{dc}, is independent of the reinforcement percentage, ρ, the corner efficiency can be calculated for different values of ρ. This is shown as a solid line in Fig. 15. Available results from other investigators are also shown. These specimens are not quite comparable, due to different concrete and reinforcement properties, dimensions, and method of loading. However, the comparison still gives a reasonable agreement for this reinforcement detail at varying percentages of tensile steel.

Development of New Corner Reinforcement Detail Adapted to Actual Stresses.—From the elastic analysis of the stress distributions in the uncracked range (Figs. 2 and 3), as well as from the test results aforementioned, it is obvious that a corner has two weak regions. One is the inside of the corner, where a peak stress, σ_x, gives early cracking and high stresses in the reinforcement. The other is the outside of the corner, where diagonal tensile stresses, σ_y, give rise to a diagonal crack and a tendency to push off the outside of the corner.

TABLE 2.—Outline of Corner Details Tested

Specimen number (1)		Haunch size, H, in inches (2)	Concrete Strength, in pounds per square inch f_c' (3)	f_{sp} (4)	Failure moment, M_{ut}, in foot-pounds (5)	Calculated ultimate moment, M_{uc}, in foot-pounds (6)	M_{ut}/M_{uc}, as a percentage (7)	Corner crack width at 55% M_{uc} (working load), in inches (8)
	U 21		4,820	390	7,160	22,680	32	Failed
	U 27		3,940	300	13,310	21,630	61	0.24
	U 15		4,110	365	16,110	23,800	68	0.28
	U 12		4,760	445	17,890	23,290	77	0.11
	U 28		3,870	290	18,370	23,040	79	0.10
	UV 1 UV 2	5.9 2.0	4,340 4,910	335 360	22,860 22,570			
	U 24		5,660	370	20,280	23,430	87	0.13
	UV 3 UV 4	3.9 2.0	4,520 3,940	345 315	26,850 25,350			0.03 0.02
	UV 5 UV 6 UV 7		4,760 4,150 4,820	375 345 260	26,250 25,350 27,290	23,000 21,990 22,210	114 115 123	0.04 0.05 0.05

Note: 1 psi = 6.9 kN/m²; 1 in. = 25.4 mm; 1 ft-lb = 1.36 N · m.

In order to study the possibilities of strengthening the inner corner with a haunch and extra diagonal reinforcement, specimens UV1 and UV2 were tested (see Table 2). According to the theory of elasticity, as the size of the haunch is increased, the magnitude of the bending stresses, σ_x, and the radial stresses, σ_y, decreases. On loading to failure, there were small cracks on the inner parts of the corner subject to tension, while the radial stresses, σ_y, caused the outside portion of the corner, which was not covered by the reinforcement, to be pushed off.

To further improve the placing of the reinforcement, it must be shaped in such a way as to prevent separation of the outside of the corner. In specimen UV3, the main reinforcement was drawn from the inside tension face out into the corner and back again into the compressed zone of the same cross section. At loading, failure occurred at an adjacent section outside the corner. In the following test series, this corner detail is systematically studied.

Systematic Study of Developed Corner Detail.—In one test series the influence of the haunch size was studied (specimens UV3, UV4, and UV5). It was found possible to get sufficient corner strength without a haunch. In another test series (specimens UV5, UV6, and UV7), the area of least inclined reinforcement for a corner without a haunch was determined by studying the effects of varying the area of the inclined reinforcement. It was determined experimentally that the area of the inclined reinforcement should be about one-half the main reinforcement area to attain yield and at the same time cause failure to occur at a section outside the corner part (6).

Function of Reinforcement at Working Load.—At working load a crack pattern, as shown in Fig. 16, can be assumed in the corner, with one diagonal crack inside and one outside the reinforcement bends. As a consequence of the crack pattern, the compression zone is located between the diagonal cracks inside the bends.

The bent portions, which provide anchorage of the main reinforcement, carry the tensile stresses out into the compression zones of the corner, and are transferred through bond and contact pressure with the concrete. The bends have the effect of closing the diagonal crack and binding the sections together, thus contributing effectively to the resistance of the radial stresses, σ_y, in the corner.

Function of Reinforcement at Failure Stage.—When failure of the corner takes place, a crack pattern, such as the one shown in Fig. 17, is formed. The cracks in the corner run in the diagonal direction into the compression zones parallel to the inclined reinforcement. The occurrence of these second diagonal cracks causes failure of the corner. The inclined reinforcement stiffens the corner and counteracts cracking.

Before the occurrence of these diagonal cracks, the forces, F_c, of the compression zones are counterbalanced by tensile forces F_s in the reinforcing bars and tensile stresses in the concrete. The occurrence of these diagonal tension cracks parallel to the inclined reinforcement causes failure. It is not possible to reinforce all tensile stresses that occur in so simple a way. Therefore, the ultimate flexural strength of the corner depends on the tensile strength of the concrete. However, there is a reinforcement percentage limit below which corner failure does not occur in this way. This limit is determined experimentally for some steel yield stresses and corner angles.

Corner Deformations.—Deformations in corners can be very large if the reinforcement layout is unsuitable. Using the proposed layout with the inclined reinforcement area one-half the main tensile reinforcement area, corner deformations have little influence on frame analysis and may normally be ignored. This is mainly due to the fact that the inclined reinforcement stiffens the corner.

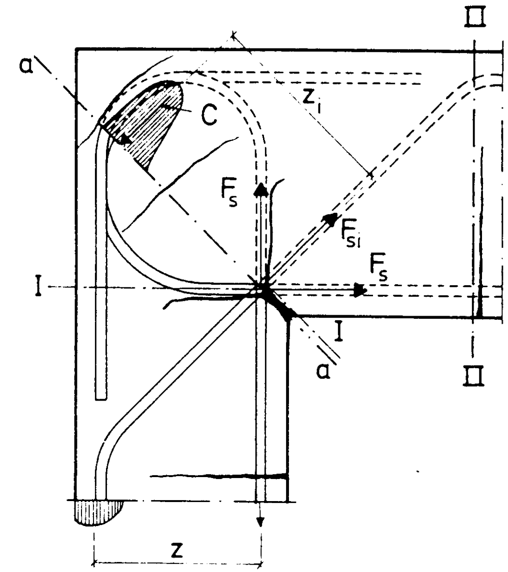


FIG. 16.—Internal Force System in Developed Corner Reinforcement Detail at Working Load

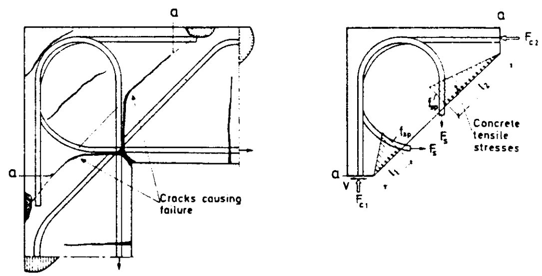


FIG. 17.—Crack Pattern and Forces Acting at Failure Stage

Outline of Details Tested.—Table 2 summarizes the types of corner reinforcement details studied. The test specimens are directly comparable since they have the same dimensions and the same quality of reinforcement [Swedish deformed bars Ks40, $f_y = 57{,}000$ psi (390,000 kN/m^2)], and the concrete compressive strength is approx 4,300 psi (30,000 kN/m^2). The tests listed in the table were carried out with a main reinforcement percentage of 0.75%.

At higher reinforcement percentages the efficiency reduces and the corner crack widths increase. In a complementary test series, the proposed corner reinforcement detail and some alternatives to this basic idea were studied further.

TABLE 3.—Test Results, 135° Corners

Specimen number (1)	Concrete Strength, in pounds per square inch f_c (2)	Concrete Strength, in pounds per square inch f_{sp} (3)	Diameters, in inches Main bars (4)	Diameters, in inches Inclined bars (5)	Measured failure load, P, in pounds (6)	M_{ut}/M_{uc}, as a percentage (7)
V1	5,220	360	0.47	None	3,935	49
V2	4,740	350	0.47	None	7,055	88
V3	5,530	385	0.47	0.39	10,915	134
V4	7,280	350	0.47	0.31	11,355	121

Note: 1 lb = 0.453 kg; 1 psi = 6.9 kN/m²; 1 in. = 25.4 mm.

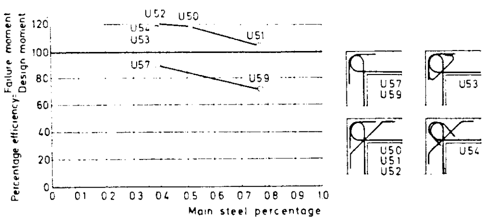


FIG. 18.—Corner Efficiency M_{ut}/M_{uc} as Function of Reinforcement Percentage ρ for Corners Reinforced with Ks60

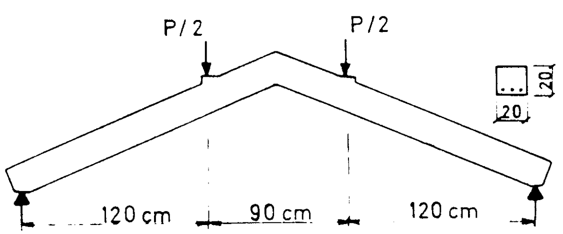


FIG. 19.—Dimensions and Method of Loading of Test Specimen (1 in. = 25.4 mm)

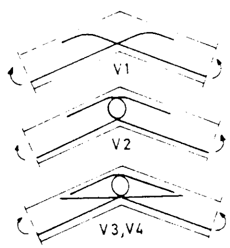


FIG. 20.—Corner Reinforcement Details

These specimens were reinforced with Ks60 bars [f_y = 85,000 psi (590,000 kN/m²)]. Fig. 18 shows the corner efficiencies obtained as functions of the main steel percentage.

Obtuse-Angled and Acute Angled Corner.—Corners with obtuse and acute angles occur on bridge abutments between the wing walls and the front wall. In order to investigate 135° corners, V-shaped beam (see Fig. 19) was used as test specimen. Fig. 20 shows the reinforcement details studied.

Table 3 summarizes the test results. The specimens are directly comparable because they have the same dimensions and the same strength of reinforcement [Swedish deformed bars Ks40, f_y = 57,000 psi (390,000 kN/m²)]. The reason for the high values of failure moment to design moment for specimens V3 and V4 is that the beam sections are calculated as singly reinforced as the influence of the inclined reinforcement is ignored (6,7).

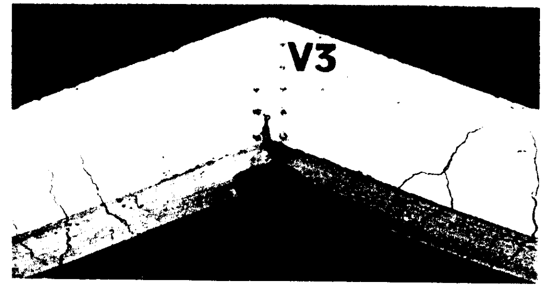


FIG. 21.—Bending Failure in Beams while Corner Remained Intact

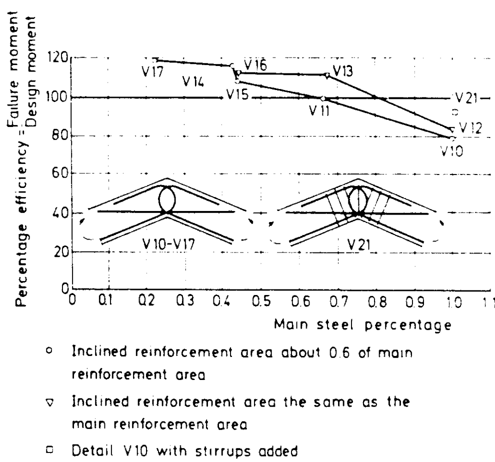


FIG. 22.—Efficiency of 135° Corners as Function of Main Steel Percentage

When loading specimen V1, a diagonal crack, which ran along the bent reinforcement, occurred in the corner and the outside of the corner was pushed off. Failure occurred suddenly and the ultimate load capacity was dependent on the tensile strength of the concrete.

Reinforcement detailing according to specimen V2 prevented the corner from being pushed off. The loops tightened and restrained the splitting force across the corner diagonal. However, a wide corner tension crack occurred during loading, and the specimen failed. To reduce the corner crack width and increase the corner strength, the reinforcement was augmented by inclined bars as for specimen V3. On loading this specimen, wide bending cracks occurred in the beams while the corner remained intact (see Fig. 21).

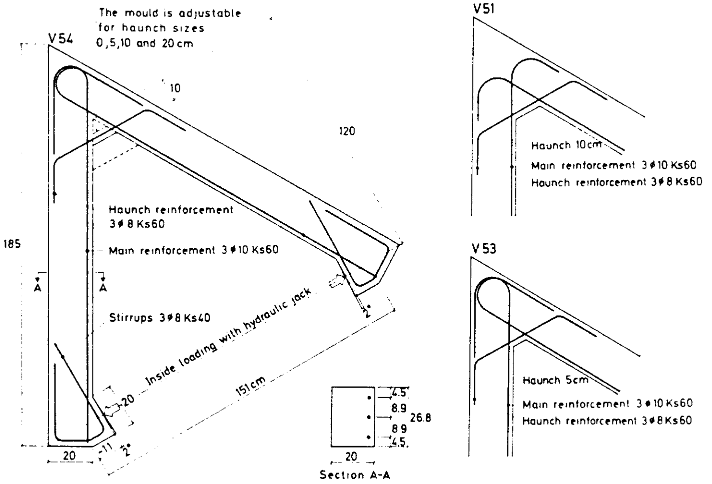


FIG. 23.—Reinforcement Details for 60° Corners—Dimensions and Method of Loading of Test Specimen (1 in. = 25.4 mm)

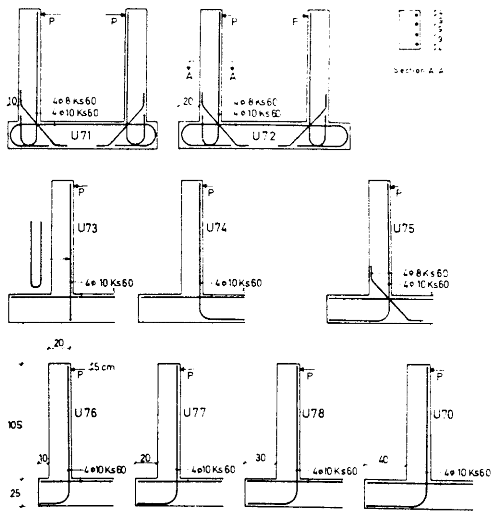


FIG. 24.—Test Program for Retaining Wall Joints (1 in. = 25.4 mm)

The corner reinforcement detail, developed as for specimen V3, has been studied in another nine tests using Swedish deformed bars Ks60 [f_y = 85,000 psi (590,000 kN/m^2)] and a concrete cube strength of approx 4,300 psi (30,000

TABLE 4.—Test Results, Joints in Retaining Walls

Specimen number (1)	Toe length, in inches (2)	Concrete Strength, in pounds per square inch f_c (3)	Concrete Strength, in pounds per square inch f_{sp} (4)	Corner crack width at 55% M_{uc} (working load), in inches (5)	M_{ut} tested, in foot-pounds (6)	M_{uc} calculated, in foot-pounds (7)	M_{ut}/M_{uc}, as a percentage (8)
U71	3.9	7,750	525	0.071	27,410	24,190	113
U72	7.9	7,440	545	0.043	27,490	24,230	113
U73	15.8	4,280	280	0.165	17,870	25,100	71
U74	15.8	6,040	250	0.185	13,890	23,110	60
U76	3.9	4,790	290	0.256	24,340	25,860	94
U77	7.9	4,690	330	0.208	25,500	25,820	99
U78	11.8	4,660	280	0.208	24,190	25,820	94
U70	15.8	3,730	380	0.216	23,580	23,360	101
U75	15.8	4,990	180	0.051	26,760	22,420	119

Note: 1 psi = 6.9 kN/m^2; 1 in. = 25.4 mm; 1 ft-lb = 1.36 N · m.

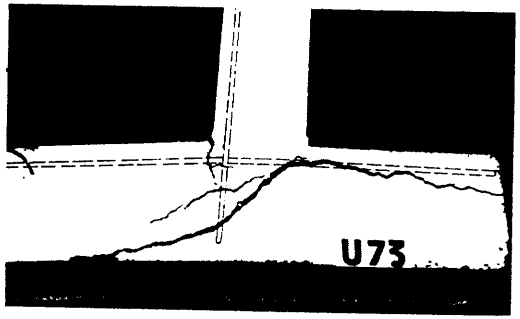


FIG. 25.—Diagonal Tension Crack Failure in Joint Caused by Bending Moment (Toe Was Pushed Off at Failure)

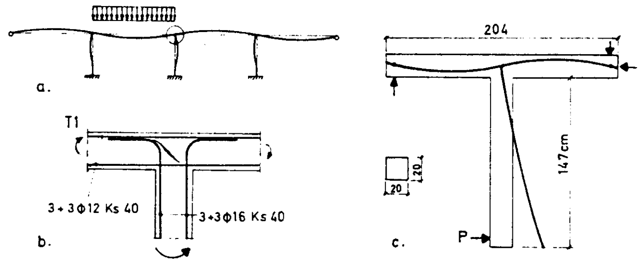


FIG. 26(*a*).—Deflected Shape of Bridge with Load on One Span: (*b*) Reinforcement of T-Joint According to Earlier Practice; (*c*) Dimensions and Method of Loading of Test Specimen (1 in. = 25.4 mm)

kN/m²). In these tests the influence of reinforcement percentage, the area of inclined reinforcement, and the stirrups in the corner were studied. Fig. 22 shows the corner efficiencies obtained (8).

The behavior of 60° corners subjected to positive moment was investigated with three specimens (see Fig. 23) (4). When loading specimen V51, diagonal tension crack failure occurred. The failure had the same characteristics as in the tests with 90° and 135° corners, and occurred at half of the intended ultimate load. Corner reinforcement for specimens V53 and V54 was arranged according to the technique developed for 90° corners. The inclined reinforcement was laid across a haunch. When specimens V53 and V54 were loaded, failure occurred in the beams outside the corners.

Joints in Retaining Walls

At the junction between the stem and base slab of retaining walls, corners are subjected to tensile stresses on the inside. These corners are similar to corners subjected to positive moments. To study the reinforcement detailing in structures of this kind, a series of tests were carried out with specimens according to Fig. 24. The specimen used in earlier corner tests had toes of different lengths added. The actual load of earth pressure was replaced with a simple point load. Fig. 24 shows some reinforcement details tested with the aim of developing simple and functional rules for joints in retaining walls with different toe lengths (4).

The test results are summarized in Table 4. All specimens are comparable because they had the same amount of reinforcement and the concrete cube strength was about the same. The reinforcement used was Ks60. When joints were reinforced in a way analogous to the proposal for corners subjected to positive moment, sufficient joint strengths were obtained for short toe lengths (U71 and U72). Specimens U73 and U74 are examples of unsuitable reinforcement arrangements that made the occurrence of diagonal tension crack failures possible (see Fig. 25). When the joint was reinforced according to U76, U77, U78, and U70, almost sufficient strength was obtained when the toe was long enough to provide adequate anchorage length. This reinforcement detail was characterized by large corner crack widths in loading tests. The corner crack width could be reduced by adding inclined reinforcement (U75).

T-Joints Subjected to Bending Moments

T-joints in cast-in-situ reinforced concrete structures may be divided into two main types: (1) Connections between members of small width, for instance, between a column and a beam; and (2) connections between walls and slabs where the joints have a large longitudinal dimension. In the latter case, the use of stirrups entails such a considerable constructional problem that they should be avoided.

Joints of the first type, beam-column joints, have been studied by several other investigators and will not be dealt with in this paper. An example of a T-joint of the second type is the connection between bridge slabs and the supporting piers.

Pilot Tests.—The T-joints occur in multispan bridges [see Fig. 26(*a*)]. The

method of reinforcing such joints, which was previously the practice in Sweden, is shown in Fig. 26(*b*). Specimen T1, having the dimensions shown in Fig.

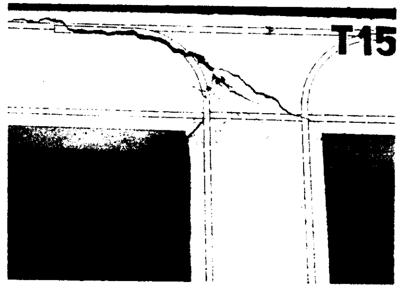


FIG. 27.—Specimen T15 after Diagonal Tension Crack Failure Caused by Bending Moment (Reinforcement Drawn on Surface)

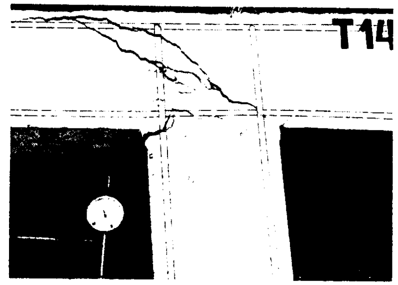


FIG. 28.—Specimen T14 after Diagonal Tension Crack Failure

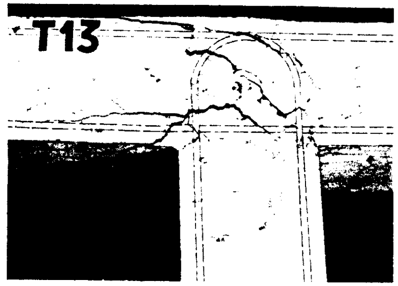


FIG. 29.—Specimen T13 after Anchorage Failure

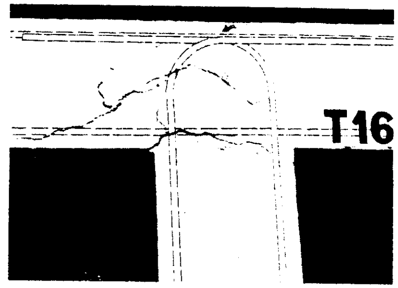


FIG. 30.—Specimen T16 after Bending Failure in Column Close to Slab

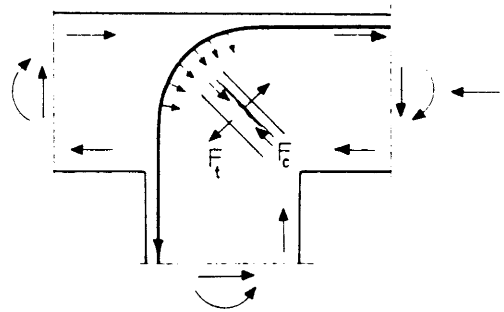


FIG. 31.—Forces in T-Joint According to Truss Mechanism

26(*c*), was tested in a pilot test in 1967. At failure a diagonal crack suddenly occurred in the joint. The ultimate load in this case was only one-third of that calculated. The consequence of this reinforcement layout is that the moment distribution in the frame will not be the one intended and designed. After the joint fails, the restraining moment is redistributed and increases the moment in the span in adjacent slabs.

The results of the pilot study indicated that other joint reinforcement details should be investigated.

Tests on Some Different Joint Details.—In the four simple reinforcement details

TABLE 5.—Diagonal Cracking Moment

Specimen (1)	Concrete tensile strength, f_{sp}, in pounds per square inch (2)	M_{dc}, in foot pounds: Calculated (3)	M_{dc}, in foot pounds: Observed (4)	Observed/calculated (5)
T1	305	4,990	6080	1.22
T11	375	6,440	7880	1.22
T15	350	5,930	5060	0.85

Note: 1 psi = 6.9 kN/m².

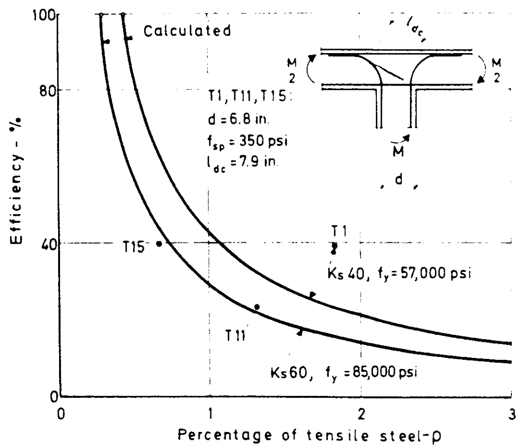


FIG. 32.—Influence of Reinforcement Percentage on Efficiency of Earlier Standard T-Joint Detail (1 in. = 25.4 mm; 1 psi = 6.9 kN/m²)

studied, the layout was successively adapted in view of the actual stresses and the results of the previous tests in the series. The joints were tested in largely the same way as in the pilot tests. The tests were carried out during 1970 (4).

At loading specimen T15, a corner crack first occurred, followed by a brittle diagonal tension crack failure caused by the bending moment (Fig. 24). The crack pattern was the same as in pilot test T1.

TABLE 6.—Outline of T-Joints Tested

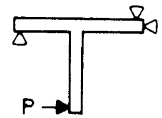
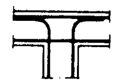
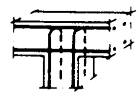
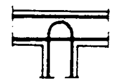
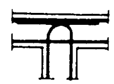
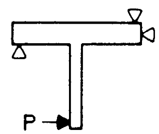
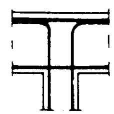
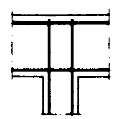
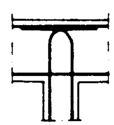
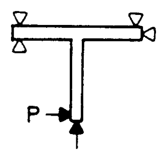
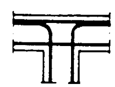
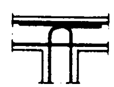
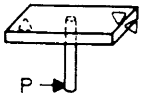
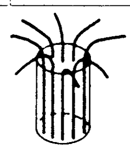
Way of loading (1)	Specimen number (2)		Reinforcement percentage in leg, ρ, as a percentage (3)	Concrete Strength, in pounds per square inch: f_c (4)	Concrete Strength, in pounds per square inch: f_{sp} (5)	M_{ut} tested, in foot-pounds (6)	M_{uc} calculated, in foot-pounds (7)	M_{ut}/M_{uc}, as a percentage (8)
		T 1	1.80	4,380	305	10,380	26,870	39
		T 15	0.65	5,380	350	7,230	17,900	40
		T 11	1.30	5,260	375	8,030	33,890	24
		T 14	0.65	4,540	325	10,310	17,720	58
		T 13	1.30	4,590	400	26,580	33,490	79
		T 2	1.80	3,380	265	29,000	26,400	110
		T 12b	1.30	4,410	350	28,350	34,390	82
		T 16	0.65	5,420	380	17,830	19,020	104
		T 25	1.81	3,980	315	16,640	41,550	40
		T 26	1.81	4,830	355	34,250	42,600	80
		T 27	1.81	5,080	385	36,890	42,710	86
		T 21	1.30	4,880	295	10,490	34,940	30
		T 22	1.30	4,350	405	32,770	34,860	94

TABLE 6.—*Continued*

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	 T 31	—	5,520	475	12,010	10,780	111
	 T 32	—	5,520	475	11,650	10,920	107

Note: 1 psi = 6.9 kN/m²; 1 ft-lb = 1.36 N · m.

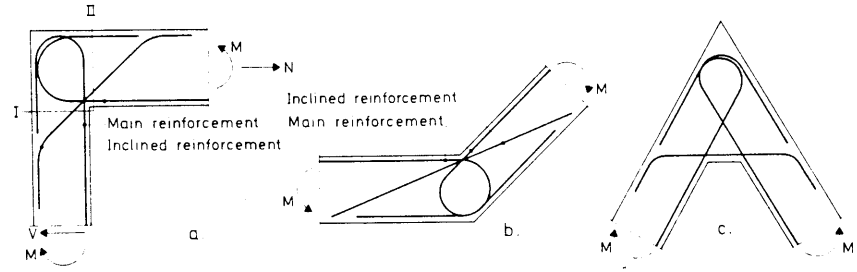


FIG. 33.—Layout of Reinforcement in Right-Angled, Obtuse-Angled, and Acute-Angled Frame Corners Subjected to Positive Moment

The reinforcement in specimen T14 consisted of hairpins that were perpendicular to the plane of the picture and were anchored at the upper face of the slab. Diagonal tension crack failure also occurred in this case (see Fig. 28).

In specimen T13 the hairpins were turned 90° and were thus in the plane of the photo (see Fig. 29). The reinforcement loop was thereby placed in the direction of the diagonal stresses and perpendicular to the diagonal crack. The ultimate load was increased as expected but not enough, since there was a certain slip along the loop due to insufficient anchorage.

The inadequacy of anchorage in T13 was remedied in specimen T16 in which the reinforcement consisted of two portions (see Fig. 30) and therefore was adequately anchored. The reinforcement layout also counteracted the occurrence of a diagonal crack in the center of the joint. The primary reason for failure was a wide flexural crack in the column close to the slab. Secondly, a diagonal crack occurred in the center of the joint after considerable deformations.

The forces acting in a T-joint, reinforced as in specimen T16, are shown in Fig. 31. The bent tension bar fosters contact pressure under the bend, which is balanced by a diagonal compression strut with the compressive force, F_c. Diagonal tension stresses σ_y in the joint may cause a diagonal crack (see Fig. 30).

Application of Diagonal Tension Crack Failure Analysis.—In T-joint details reinforced according to specimens T1, T15, and T14, diagonal tension crack failures caused by bending moments developed. The moment, M_{dc}, at which

diagonal tension crack failure occurs was derived with the help of a truss analogy (see Fig. 6 and Eq. 6). This equation will now be applied to the tested joints that correspond to this mode of failure.

The position of the diagonal tension crack and values for calculation of M_{dc} are given in Fig. 32. The length of the tensile stress distribution before cracking, l_{dc}, is estimated by the theory of elasticity (Fig. 4). Table 5 summarizes the calculated and observed values of M_{dc}. The agreement is rather good.

An expression for the reinforcement percentage, ρ, for which the reinforcement with this type of detailing will yield before the diagonal crack occurs, can be derived in the same way as for corners subjected to positive moment (compare Eqs. 7-9)

$$\rho \leq \frac{4}{3\sqrt{5}} \frac{f_{sp}}{f_y} \frac{l_{dc}}{d} \qquad (10)$$

When Eq. 10 is applied to T-joints with dimensions, concrete, and steel yield stresses using the test results given in this section, it is found that diagonal tension cracking failure does not occur if $\rho \leq 0.29\%$ for Ks60 and $\rho \leq 0.43\%$ for Ks40 reinforcement.

These calculated values and available test results of previous standard T-joint detail are shown in Fig. 32. Based on the assumption that the diagonal cracking failure moment, M_{dc}, is independent of the reinforcement percentage, ρ, the joint efficiency can be calculated for different values of ρ. The agreement with available test results is good.

Outline of T-Joints Tested.—Table 6 summarizes the T-joints tested. Details T1 and T2 were reinforced with Ks40 [f_y = 57,000 psi (390,000 kN/m²)] and the others with Ks60 [f_y = 85,000 psi (590,000 kN/m²)].

Reinforcement Proposals

The following proposals for design rules have been developed for the reinforcement of corners, subjected to positive moment, retaining wall corners, and T-joints, as a result of the work performed.

Frame corners subjected to a positive moment, M, according to Fig. 33 are to be detailed according to the design rules given in the following paragraphs. The reinforcement loop from each adjacent part of the structure is taken out into the corner region as far as requirements regarding cover allow and is then brought back into the same cross section to the level of the inclined reinforcement. The main reinforcement is designed on the basis of the moments and normal forces in the adjacent Sections I and II, ignoring the effect of reinforcement loop curtailment in the compression zone and the inclined reinforcement. The cross-sectional area of the inclined reinforcement is approximately one-half of the largest main reinforcement area.

The requirement regarding the least permissible bending radius and the spacing of reinforcing bars generally results in the dimensions of structural elements being limited. The dimensions of the cross section are also chosen in such a way that the following restrictions of the reinforcement percentage, ρ, are satisfied in order to avoid failure in the corner. The rules are specially given for Swedish deformed bars Ks40 and Ks60 with yield stresses of 57,000 psi (390,000 kN/m²) and 85,000 psi (590,000 kN/m²), respectively. The restrictions are based on tests using concrete with a cube strength of 4,300 psi (30,000 kN/m^2). For Grade Ks40: at 90°, $\rho \leq 1.2\%$ and at 135°, $\rho \leq 1.0\%$. For Grade Ks60: at 90°, $\rho \leq 0.8\%$ and at 135° $\rho \leq 0.65\%$. The aforementioned values of maximum reinforcement percentage may be interpolated for intermediate angles and interpolated or extrapolated with regard to the yield stress of other steel grades.

If the area of the inclined reinforcement in a 135° corner is the same as that of the main reinforcement, the reinforcement percentage, ρ, may be increased to a maximum of 1.2% for Ks40 and 0.8% for Ks60. If the reinforcement percentage is higher than this, the corner must be provided with a reinforced haunch and stirrups.

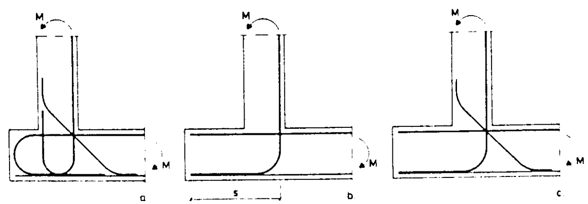


FIG. 34.—Layout of Reinforcement in Retaining Wall Corners

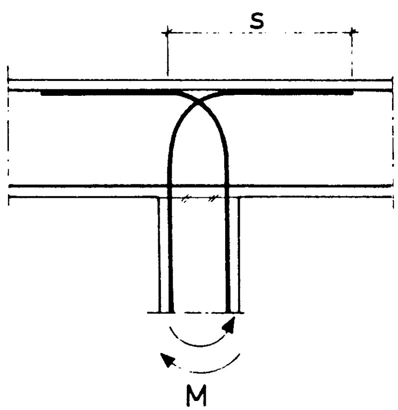


FIG. 35.—Layout of Reinforcement in T-Joint

In acute-angled corners the inclined reinforcement is laid in a haunch [see Fig. 33(c)]. The length of this haunch is at least one-half the thickness of the wing wall when the reinforcement percentage, ρ, is less than 0.75% for Ks40 and 0.5% for Ks60, and at least equal to the thickness of the wing wall when the reinforcement percentage, ρ, is less than 1.2% for Ks40 and 0.8% for Ks60.

Reinforcement for a negative corner moment is taken out into the corner as far as bending regulations permit. Bars must not be spliced in the corner region.

Corners in retaining walls are to be reinforced according to the general solutions

[see Fig. 34(a), 34(b), and 34(c)] given in the following paragraphs.

When the length of the toe is less than the stem thickness, the joint is reinforced as a corner subjected to positive moment. The reinforcement in the base slab is taken out into the toe as far as the cover requirement permits.

When the length of the toe is greater than the stem thickness and the length of the toe is sufficient to provide adequate anchorage length s, reinforcement is as shown in Fig. 34(b). In the requirements for concrete regulations regarding bending radius, spacing of bent bars and cover must be borne in mind.

The reinforcement layout according to Fig. 34(b) results in a wide corner crack in the angle subjected to tension. If limitation of the corner crack width is essential, the reinforcement should be augmented by inclined reinforcement as in Fig. 34(c).

Walls and slabs which join one another in the form of T-joints subjected to a bending moment are to be reinforced according to Fig. 35. The usual anchorage rules are to be applied with respect to the adjoining reinforcement. The reinforcement percentage should not exceed 1.8% for Ks40 and 1.2% for Ks60 if the joint is to have the intended moment capacity. The spacing of the bars is to be such that concreting is possible.

Recesses or holes should not be made at or in the immediate vicinity of corners or joints, since these considerably reduce the strength and stiffness of the connection. By using the aforementioned solutions for the design and detailing of corners and joints, their capacity to resist moments and forces from adjacent members of the structure will be such that the load-bearing capacity of the adjacent cross sections is utilized fully. Cracks in the corners are limited to acceptable widths. The proposals for design rules are based on tests with monotonic one-way loading to failure. Tests with cyclic loading positive-negative moment and large deformations after yielding (earthquake loadings) are not included in this investigation.

Appendix.—References

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